Deterministic and stochastic acceleration techniques for Richardson-type iterations

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Introduction

- Scientific computing is moving to exascale
- Applications require high level of concurrency
- Next generation computers will exhibit more hardware failures – applications must be resilient
- Standard Krylov subspace methods struggle to simultaneously obtain efficiency and concurrency
- Standard Krylov methods struggle to achieve resilience
- Richardson's schemes benefit computational and data locality

Mathematical framework

- Reformulate the sparse linear system of interest

$$A\mathbf{x} = \mathbf{b}$$

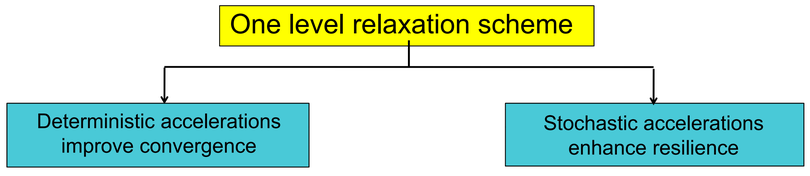
as a fixed point scheme

$$\mathbf{x} = H\mathbf{x} + \mathbf{f}$$

such that the Neumann series recasts the solution as

$$\mathbf{x} = \sum_{i=0}^{\infty} H^i \mathbf{f} \quad (1)$$

- One level fixed point schemes are renown for their deteriorated asymptotic convergence rate
- Multilevel schemes can accelerate convergence
- Purely deterministic algorithms struggle to deal with inherently random faulty phenomena



Alternating Anderson-Richardson (AAR)

AAR uses approximate solutions computed by successive Richardson's steps to build the matrix

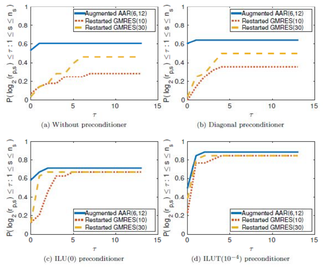
$$U_k = [-(\mathbf{x}^{k-p+1} - \mathbf{x}^{k-p}), \cdots, -(\mathbf{x}^{k} - \mathbf{x}^{k-1})].$$

The ***Anderson mixing*** is defined as

$$\overline{\mathbf{x}}^k = \mathbf{x}^k + U_k \mathbf{g}_k.$$

The vector $\mathbf{g}_k$ is chosen so as to minimize the residual norm.

- Multiple Richardson's steps without optimization benefit computational and data locality
- Convergence on ***positive definite matrices*** guaranteed



Conclusions and future developments

- Identified classes of matrices for which convergence is guaranteed
- Competitive performance compared to restarted GMRES for different choices of preconditioner
- ➢ Convergence analysis on specific classes of problems
- ➢ Performance assessment at extreme scale
- ➢ Use of GPU accelerations

References

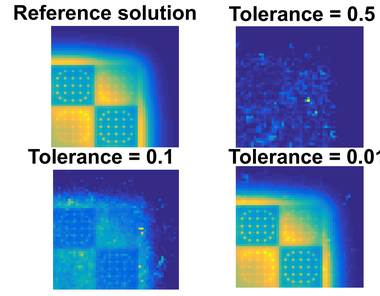
• "Anderson acceleration of the Jacobi iterative method: an efficient alternative to Krylov methods for large, sparse linear systems", P. P. Pratapa, P. Suryanarayana, and J. E. Pask – J. Comput. Phys., 306, pp. 4354, 2016.
• "Converge analysis of Anderson-type acceleration for Richardson's iteration", M. Lupo Pasini – In preparation.

Monte Carlo Linear Solvers (MCLS)

MCLS use random walks defined on a transition matrix ***P*** and a sequence of weights $w_{i,j} = H_{i,j} / P_{i,j}$, the statistical estimator redefines (1) as

$$\theta = x_i = f_i + \sum_{l=1}^{\infty} \sum_{k_1=1}^{n} \cdots \sum_{k_\ell=1}^{n} P_{i,k_1} \cdots P_{k_{\ell-1},k_\ell} w_{i,k_1} \cdots w_{k_{\ell-1},k_\ell} f_{k_\ell}.$$

- Convergence guaranteed for ***strictly diagonally dominant***, ***M matrices***, ***generalized diagonally dominant matrices***
- Choice of preconditioners affects convergence
- We use adaptive methods to select the number of histories



Conclusions and future developments

- Identified classes of matrices and preconditioners for which MCLS are guaranteed to converge a priori
- Difficulty in ensuring a priori convergence of MCLS for a general problem
- ➢ Algorithm scalability has not yet been analyzed
- ➢ Testing MCLS on large parallel architectures and evaluating their resilience in presence of faults still to be addressed

References

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